MVM -experience the excellence-

PLEASURE TEST REVISION SERIES XII
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HOMEWORK TEST – 01

Time Allowed: 180 Minutes **Max. Marks: 100**

General instructions:

a) **Note that all the questions are compulsory.**

b) **The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each, and Section C comprises of 7 questions of six marks each. All questions in section A are to be answered in one word, one sentence or as per the exact requirements of the question.**

c) **There is no overall choice. However internal choice has been provided in some of the cases.**

[SECTION – A]

Q01. Find the direction cosines of the line: $x = 2 - 2p,\ y = 3 + p,\ z = 4 - 5p$.

Q02. Write the value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$.

Q03. Evaluate: $\int_0^3 f(x)dx$, where $f(x) = \log\left(\frac{3}{x} - 1\right)$.

Q04. Determine the value of x and y in the matrix equation: $\begin{pmatrix} 3 \\ 2 \end{pmatrix}\begin{pmatrix} 1 & 5 \end{pmatrix} = \begin{pmatrix} 3 & 7x+y \\ 2y & 10 \end{pmatrix}$.

Q05. Find the values of α and β such that $\int \frac{dx}{1+\sin x} = \tan\left(\frac{x}{2} + \alpha\right) + \beta$.

Q06. For what value of m and p is the matrix $\begin{pmatrix} 0 & 5 & -3 \\ -5 & m & 4 \\ p & -4 & 0 \end{pmatrix}$ skew symmetric?

Q07. Let $\vec{a}$ and $\vec{b}$ are non-collinear vectors. For what value of x, the vectors $\vec{c} = (x-2)\vec{a} + \vec{b}$ and $\vec{d} = (2x+1)\vec{a} - \vec{b}$ are collinear?

Q08. Write the tangent of the angle between $\vec{a}$ and $\vec{b}$ such that $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$.

Q09. Write the number of binary operations that can be defined on the set $\{1, 2\}$.

Q10. Write a unit vector parallel to '$-\vec{a}$' if it is given that $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$.

[SECTION – B]

Q11. For what value(s) of 'a' and 'b', the function $f(x) = \begin{cases} 3ax+b, & if\ x > 1 \\ 11, & if\ x = 1 \\ 5ax-2b, & if\ x < 1 \end{cases}$ is continuous at $x = 1$?

Q12. Show that: $\tan^{-1}\left(\frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}}\right) = \frac{\pi}{4} - \frac{x}{2},\ x \in \left(\pi, \frac{3\pi}{2}\right)$.

(OR) Show that: $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right) = \cos^{-1}\left(\frac{a\cos\theta + b}{a + b\cos\theta}\right)$.

Q13. Evaluate the integral: $\int \frac{\sin 2x \cos 2x}{\sqrt{9 - \cos^4 2x}}dx$.

Q14. Solve the differential equation: $\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0$.

Q15. If $x=\sin t\sqrt{\cos 2t}$, $y=\cos t\sqrt{\sin 2t}$ then, find $\frac{dy}{dx}$. Also find $\left.\frac{dy}{dx}\right]_{at\ t=\frac{\pi}{8}}$.

Q16. Tanu and Manu throw a die alternatively till one of them gets a 'six' and wins the game. Find their respective probabilities of winning, if Tanu starts the game. What is the importance of outdoor games in life?

Q17. Water is dripping out from a conical funnel at a uniform rate of $4cm^3/s$ through a tiny hole at the vertex in the bottom. When the slant height of the water is 3cm, find the rate of decrease of the slant height of the water-cone. Given that the vertical angle of funnel is 120°.

(OR) Show that $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y=be^{-x/a}$ at the point where it crosses y-axis.

Q18. Consider $f:\mathrm{R}_+\to[-5,\infty)$ given by $f(x)=9x^2+6x-5$. Find the expression for f^{-1}.

Q19. Evaluate: $\int x\sqrt{\frac{16-x^2}{16+x^2}}dx$. **(OR)** Evaluate: $\int\sqrt{\frac{1-5x}{1+5x}}dx$.

Q20. Using properties of determinants, prove that: $\begin{vmatrix}(y+z)^2 & xy & zx\\ xy & (x+z)^2 & yz\\ xz & yz & (x+y)^2\end{vmatrix}=2xyz(x+y+z)^3$.

Q21. Find the equation of a plane that contains $(1,-1,2)$ and is perpendicular to each of the planes $2x+3y-2z-5=0$ and $x+2y-3z-8=0$.

(OR) Find the equation of a plane which contains the line of intersection of the planes $\vec{r}.(\hat{i}+2\hat{j}+3\hat{k})=4$, $\vec{r}.(2\hat{i}+\hat{j}-\hat{k})+5=0$ and which is perpendicular to plane $\vec{r}.(5\hat{i}+3\hat{j}-6\hat{k})+8=0$.

Q22. If $\vec{\alpha}=3\hat{i}-\hat{j}$ and $\vec{\beta}=2\hat{i}+\hat{j}-3\hat{k}$, then express $\vec{\beta}$ in the form of $\vec{\beta}=\vec{\beta}_1+\vec{\beta}_2$ where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

[SECTION – C]

Q23. Solve the system of equations using matrix: $3x+\frac{4}{y}+7xz=14,\ 2x-\frac{1}{y}+3xz=4,\ x+\frac{2}{y}-3xz=0$.

Q24. Show that height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi-vertical angle 60° is one-third that of the cone and the greatest volume of cylinder is given as: $\frac{4}{9}\pi h^3$.

(OR) An open box with the square base is to be made out of a given quantity of card-board of area A^2 square units. Show that the maximum volume of box is $\frac{\mathrm{A}^3}{6\sqrt{3}}$ cubic units.

Q25. If a machine is correctly set up, it produces 90% acceptable items. If it is incorrectly set up, it produces only 40% acceptable items. Past experience shows that 80% of the set ups are correctly done. If after a certain set up, the machine produces two acceptable items then, find the probability that the machine is correctly setup. 'Machines are our friends.' Comment.

Q26. Find the equation of perpendicular from the point $(1,6,3)$ to the line $x=\frac{y-1}{2}=\frac{z-2}{3}$. Also find the length of perpendicular from the point $(1,6,3)$.

Q27. Using integration, find the area of the region bounded by $\left\{(x,y): |x-1|\le y\le\sqrt{5-x^2}\right\}$.

(OR) Find the area bounded by $x^2+y^2=25,\ 4y=|4-x^2|,\ x=0$ which is lying above x-axis.

Q28. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain atleast 8 units of vitamin A and 10 units of vitamin C. Food X contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food Y contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹50 per kg to purchase Food X and ₹70 per kg to purchase Food Y. Formulate this problem as a linear programming problem to minimise the cost of such a mixture.

Q29. Prove that: $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$. Hence evaluate: $\int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$.

[ANSWERS OF PLEASURE (HOMEWORK) TEST – 01]

Q01. $-\frac{2}{\sqrt{30}},\frac{1}{\sqrt{30}},-\frac{5}{\sqrt{30}}$ **Q02.** $\frac{\sqrt{3}}{2}$ **Q03.** 0 **Q04.** $x=2,y=1$

Q05. $\alpha=-\frac{\pi}{4},\beta=$ Integral constant **Q06.** $p=3,m=0$ **Q07.** $\frac{1}{3}$ **Q08.** $\frac{\pi}{4}$

Q09. 16 **Q10.** $\frac{1}{7}\left(-3\hat{i}+2\hat{j}-6\hat{k}\right)$ **Q11.** $a=3,\ b=2$

Q13. $-\frac{1}{4}\sin^{-1}\left(\frac{\cos^2 2x}{3}\right)+k$ **Q14.** $\sqrt{1+y^2}+\sqrt{1+x^2}+\log\left|\sqrt{1+x^2}-1\right|=\log|x|+k$

Q15. $\sqrt{\cot 2t},1$

Q16. For Tanu, probability of winning $=\frac{6}{11}$; for Manu, probability of winning $=\frac{5}{11}$ **Q17.** $\frac{32}{27\pi}cm/s$

Q18. $f^{-1}=\frac{\sqrt{6+x}-1}{3}$ **Q19.** $8\sin^{-1}\left(\frac{x^2}{16}\right)+\frac{1}{2}\sqrt{256-x^4}+k$ **OR** $\frac{1}{5}\left[\sin^{-1}5x+\sqrt{1-25x^2}\right]+k$

Q21. $5x-4y-z=7$ **OR** $\vec{r}.(33\hat{i}+45\hat{j}+50\hat{k})=41$ **Q22.** $\vec{\beta}_1=\frac{3}{2}\hat{i}-\frac{1}{2}\hat{j};\ \vec{\beta}_2=\frac{1}{2}\hat{i}+\frac{3}{2}\hat{j}-3\hat{k}$

Q23. $x=1,y=1,z=1$ **Q25.** 0.95

Q26. Eq. of perpendicular: $\frac{x-1}{0}=\frac{y-6}{-3}=\frac{z-3}{2}$; Length of perpendicular: $\sqrt{13}\,units$

Q27. $\frac{1}{2}\left(\frac{5\pi}{2}-1\right)sq.units$ **OR** $2+\frac{25}{2}\sin^{-1}\left(\frac{4}{5}\right)sq.units$

Q28. Amount of food X: $2kg$; Amount of food Y: $4kg$; Minimum cost: ₹ 380 **Q29.** $\frac{\pi}{12}$.

WISDOM

All the people dream but not equally. Those who dream by night in the dusty recesses of their minds, wake in the day to find that it was vanity; but the dreamers of the day are dangerous, for they may act their dream with open eyes to make it possible.